

Trigonometric Identities

Name _____

Class _____

Date _____

ANSWERS

1a _____

1b _____

2a _____

2b _____

3a _____

3b _____

4a _____

4b _____

5a _____

5b _____

6a _____

6b _____

7a _____

7b _____

Verify the equations.

1. a. $\frac{(\sin \alpha + \cos \alpha)^2}{\sin 2\alpha + 1} = 1$ b. $\frac{\sin 4\alpha - 1}{(\cos 2\alpha - \sin 2\alpha)^2} = -1$

2. a. $\sin^4 \alpha + 2\sin^2 \alpha \times \cos^2 \alpha + \cos^4 \alpha = 1$

b. $1 - \operatorname{tg}^2 \alpha \times \operatorname{ctg}^2 \alpha - (\sin^2 \alpha + \cos^2 \alpha) = -1$

3. a. $\frac{1 + \sin 2\alpha + \cos 2\alpha}{1 + \sin 2\alpha - \cos 2\alpha} = \operatorname{ctg} \alpha$

b. $\frac{\sin 2\alpha - \sin 3\alpha + \sin 4\alpha}{\cos 2\alpha - \cos 3\alpha + \cos 4\alpha} = \operatorname{tg} 3\alpha$

4. a. $\frac{\sin(\alpha + \beta) + \sin(\alpha - \beta)}{\cos \alpha \times \cos \beta} = 2\operatorname{tg} \alpha$

b. $\frac{\sin(\alpha + \beta) - \sin(\alpha - \beta)}{\cos \alpha \times \cos \beta} = 2\operatorname{tg} \beta$

5. a. $\sin \alpha + \sin(\alpha + 120^\circ) + \sin(\alpha + 240^\circ) = 0$

b. $\operatorname{tg}(\pi + \alpha) \times \operatorname{ctg}\left(\frac{3\pi}{2} - \alpha\right) + \operatorname{ctg}^2\left(\frac{3\pi}{2}\right) = 2\operatorname{tg}^2 \alpha$

6. a. $\frac{\cos \alpha - \cos 5\alpha}{\sin 5\alpha + \sin \alpha} = \operatorname{tg} 2\alpha$

b. $\frac{\sin 3\alpha - \sin \alpha}{\cos 3\alpha + \cos \alpha} = \operatorname{tg} \alpha$

7. a. $\frac{\sin^2 \alpha}{1 - \cos \alpha} - \cos \alpha = 1$

b. $\frac{\cos^2 \alpha}{1 - \sin \alpha} - \sin \alpha = 1$

A

8a

8b

9a

9b

10a

10b

11a

11b

$$8. \quad \text{a.} \quad \frac{1 - \cos \alpha + \sin 2\alpha}{\sin 2\alpha - \sin \alpha} = \operatorname{ctg} \alpha \qquad \text{b.} \quad \frac{\sin 2\alpha - \sin \alpha}{1 - \cos \alpha + \cos 2\alpha} = \operatorname{tg} \alpha$$

$$9. \quad \text{a.} \quad \frac{\sin\left(\frac{\pi}{6} + \alpha\right) + \sin\left(\frac{\pi}{6} - \alpha\right)}{\cos \alpha} = 1$$

$$\text{b.} \quad \frac{\cos\left(\frac{\pi}{6} + \alpha\right) + \cos\left(\frac{\pi}{6} - \alpha\right)}{\cos \alpha} = \sqrt{3}$$

$$10. \quad \text{a.} \quad (\operatorname{tg} \alpha + \operatorname{ctg} \alpha)^2 - (\operatorname{tg} \alpha - \operatorname{ctg} \alpha)^2 = 4$$

$$\text{b.} \quad \frac{\operatorname{tg} \alpha + \operatorname{tg} \beta}{\operatorname{tg}(\alpha + \beta)} - \frac{\operatorname{tg} \alpha - \operatorname{tg} \beta}{\operatorname{tg}(\alpha - \beta)} = 2$$

$$11. \quad \text{a.} \quad \cos\left(\frac{\pi}{3} + \alpha\right) \times \cos \alpha + \sin\left(\frac{\pi}{3} + \alpha\right) \times \sin \alpha = \frac{1}{2}$$

$$\text{b.} \quad \sin\left(\frac{\pi}{6} + \alpha\right) \times \cos \alpha - \cos\left(\frac{\pi}{6} + \alpha\right) \times \sin \alpha = \frac{1}{2}$$

A